Applied Mathematics and Statistics
Common Qualifying Examination Part B
in Computational Applied Mathematics

Spring 2013 (January)

(Closed Book Exam)

Please solve 3 out of 4 problems for full credit.
Indicate below which problems you have attempted by circling the appropriate numbers:

Part B: 1 2 3 4

NAME ______________________________________

Start each answer on its corresponding question page. Print your name, and the appropriate question number at the top of any extra pages used to answer any question. Hand in all answer pages.

Date of Exam: January 28th, 2012
Time: 11:00 AM – 12:00 PM
B1. Linearize the following nonlinear system near its critical point

\[ \begin{align*}
\dot{x} &= -y - y^3, \\
\dot{y} &= x, \quad x, y \in \mathbb{R}.
\end{align*} \]

Show that the critical point for the linear system is a center. Prove that the type of the critical point is the same for the nonlinear system by showing the existence of closed orbits around the critical point.
B2. Solve the following boundary value problem on the interval \([0, 1]\) using the method of Green’s function

\[ y'' + 4y' + 4y = f(x), \quad y(0) = a, \; y(1) = b. \]
B3. Let $R_j$ denote the $j \times j$ leading principal submatrix of a nonsingular triangular matrix $R \in \mathbb{R}^{n \times n}$, i.e., $R_j = R_{1:j,1:j}$.

a) (3 points) Show that $R_j$ is nonsingular for $1 \leq j < n$.

b) (3 points) Show that $\|R_j\|_p \leq \|R_{j+1}\|_p$ for $1 \leq j < n$ and for any $p \in [1, \infty]$.

c) (4 points) Show that $\|R_j^{-1}\|_p \leq \|R_{j+1}^{-1}\|_p$ for $1 \leq j < n$ and for any $p \in [1, \infty]$. 
B4. Let $A = xy^T$, where both $x$ and $y$ are in $\mathbb{R}^n \setminus \{0\}$.

a) (3 points) What is the rank of $A$? What is the range of $A$? What is the null space of $A$?

b) (3 points) What are the eigenvalues of $A$?

c) (4 points) Show that $\det(I + A) = 1 + x^Ty$, where $I$ is the $n \times n$ identity matrix.