Solve any three of the following four problems.

All problems are weighted equally. On this cover page write which three problems you want graded.

problems to be graded:

Name (PRINT CLEARLY), ID number
1. Let \( \{X_i, i \geq 1\} \) be a sequence of i.i.d. exponential random variables with a common parameter \( \lambda > 0 \). Let \( N \) be a random variable with the probability mass function \( P(N = k) = (1 - p)p^{k-1} \) for \( k = 1, 2, \ldots \), where \( p \in (0, 1) \) is a given constant. Find the cumulative distribution function of \( \sum_{i=1}^{N} X_i \) (clearly state the assumption you used in the calculation).

2. Let \( X \) and \( Y \) be two independent discrete random variables with the probability mass functions \( P(X = i) = (e - 1)e^{-i} \) and \( P(Y = j) = \frac{1 - j}{e - 1} \) for \( i, j = 1, 2, \ldots \). Let \( \{U_i, i \geq 1\} \) be a sequence of i.i.d. uniform random variables on \([0, 1]\). Assume the sequence \( \{U_i\} \) is independent of \( X \) and \( Y \). Define \( M = \max\{U_1, U_2, \ldots, U_Y\} \). Find the distribution of \( Z = X - M \).

3. Let \( X_1, \ldots, X_n \) be \( n \) i.i.d. continuous random variables. A record is said to have occurred at time \( k \) if \( X_k > X_i \) for all \( i = 1, \ldots, k - 1 \). Let \( N \) denote the total number of records. Find the variance of \( N \).

4. Let \( X_1, X_2 \) and \( X_3 \) be three i.i.d. random variables uniformly distributed over \([0, 1]\). What is the probability that three sticks of respective lengths \( X_1, X_2 \) and \( X_3 \) can be used to form a triangle?