AMS Qualifying Examination, January 2009
Probability Questions

1. Bob tossed a symmetric coin \((n+1)\) times independently and counted the number of tails. John did the same \(n\) times. Compute the probability that Bob counted more tails than John.

2. Let \((X,Y)\) be a bivariate random variable, where \(X\) is an exponential random variable with mean 1. Suppose that \(\text{Cov}(X,Y)=-2\), \(E[Y]=-2\), and \(\text{Var}(Y)=4\). Find the cumulative distribution function of \(Y\).

3. Let \(X_1, X_2, X_3\) be three independent uniformly distributed random variables on \([0,1]\). Compute \(P\{X_3 > X_1 + X_2\}\).

4. Let \(X\) be a random variable with \(E[X]=\mu\) and \(E(X - \mu)^4 = b\). Is it true that \(P\{|X - \mu| \geq t\} \leq b/t^4\) ? Prove your answer.