June 2005 AMS 504 Questions

1. Let $f$ be a continuous function on $[0,1]$. Evaluate the limit as $n \to \infty$ of $\int x^n f(x)\,dx$.

2. Given $c$ and $x$ such that $0 < c \leq 1$ and $x > -1$, prove that $(1+x)^c \leq 1 + cx$.

3. Let $X$ be a metric space, and let $K \subset X$ be compact. Prove that $K'$, the set consisting of limit points of $K$, is compact.