Do three out of four problems.

1. An individual has one of three genotypes called $dd$, $dD$, and $DD$, respectively, for a gene associated with disease $X$. The probability that an individual has genotype $dd$ is 0.49; the probability that an individual has genotype $dD$ is 0.42; and the probability that an individual has genotype $DD$ is 0.09. The probability that an individual with the $dd$ genotype is affected with disease $X$ is 0.10. The probability that an individual with the $dD$ genotype is affected with disease $X$ is 0.75. The probability that an individual with the $DD$ genotype is affected with disease $X$ is 0.99. What is the probability that an individual is affected with disease $X$? Given that an individual has disease $X$, what is the probability that the individual is genotype $DD$?

2. A firm wishes to reduce its backlog of transactions by hiring temporary workers. The firm is equally likely to have 2, 3, 4, or 5 temporary workers on a given day. No matter how many temporary workers there are on a given day, the numbers of transactions completed on a given day by each of these temporary workers are independent Poisson random variables with mean 32. Let $X$ denote the number of transactions completed by the temporary workers on a given day. Find $E[X]$ and $\text{var}(X)$.

3. If the random variables $X$ and $Y$ are identically distributed, not necessarily independent, with finite variance, find $\text{cov}(X + Y, X - Y)$.

4. Let $X_1, X_2, X_3$ be independent and identically distributed continuous random variables. Compute $P\{X_1 > X_2 \mid X_2 < X_3\}$. 