Solve any three of the following four problems.

All problems are weighted equally. On this cover page write which three problems you want graded.

problems to be graded:

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Name (PRINT CLEARLY), ID number
1. Let $X$ be a non-negative integer-valued random variable with probability mass function $f(k) = P(X = k)$, $k = 0, 1, 2, \ldots$. Define a function $h(r) := P(X = r | X \geq r)$. Let $\{U_i, \ i \geq 0\}$ be a sequence of i.i.d. random variables uniformly distributed over $[0, 1]$. Find the distribution of $Z := \min\{n : U_n \leq h(n)\}$.

2. Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. random variables satisfying $E[\frac{1}{X_i}] < \infty$. Let $S_m = \sum_{i=1}^{m} X_i$. For any two given positive integers $m \leq n$, find $E[\frac{S_m}{S_n}]$.

3. A clerk drops $n$ matching pairs of letters and envelopes. He then places the letters into the envelopes in a random order. Let $X$ be the number of correctly matched pairs. Find the variance of $X$.

4. Let $X$ and $Y$ be two independent exponential random variables with respective parameters $\lambda$ and $\mu$. For a given $t > 0$, find the probability $P(X \leq t < X + Y)$. 
