DOCTORAL QUALIFYING EXAMINATION
FALL 2000

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME: __________________________________________

ID#: __________________________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: September 5, 2000

Time of Exam: 3:30PM-4:30PM

Place of Exam: Social Behavioral Science Bldg., Room 2228
Let $L$ denote the vector space of $n \times n$ matrices with real coefficients. For two elements $A$ and $B$, define $\langle A, B \rangle = \text{tr}(AB^t)$, where $B^t$ denotes the transpose of $B$.

(i) Show that $\langle \cdot , \cdot \rangle$ defines an inner product (dot product) on $L$.

(ii) Let $S$ be the subspace of $L$ consisting of symmetric matrices. Determine the orthogonal complement of $S$ and prove your assertion.
4. A real-valued function $f$ defined on the real line $\mathbb{R}$ is said to be periodic if there exists a number $k > 0$ such that $f(x + k) = f(x)$ for all $x \in \mathbb{R}$. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and periodic. Prove that $f$ is bounded and uniformly continuous on $\mathbb{R}$. 