DOCTORAL QUALIFYING EXAMINATION

SPRING 2003

Probability

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1  2  3  4

NAME: ________________________________

SOCAL ID#: ________________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: January 23, 2003

Time of Exam: 1-2PM

Place of Exam: Stony Brook Union Auditorium
1. Each day an experimental animal is exposed to a certain set of stimuli designed to elicit a particular response. Let \( A_k \) be the event that the animal makes the desired response on the \( k \)th day, \( A^*_k \) be the event that the animal does not make the desired response on the \( k \)th day, and suppose that \( P(A_k \mid A^*_k) = \beta \) and \( P(A_k \mid A_k) = \alpha \), where \( 0 < \alpha < \beta \leq 1 \). Let \( p_k = P(A_k) = 1 - P(A^*_k) \).

   a. Show that \( p_{k+1} = \alpha + (\beta - \alpha)p_k \).

   b. If \( \beta = 1 \) and \( p_1 = 0 \), show that \( p_k = 1 - (1 - \alpha)^{k-1} \).

   c. Show that \( \lim_{k \to \infty} (p_k) = \frac{\alpha}{1 + \alpha - \beta} \).
2. A tea-drinking lady starts out with two boxes, each of which contains \( n \) teabags. Each time she needs a cup of tea she selects one of the two boxes at random and takes a teabag from it. What is the probability that the \((n+k)\)th teabag will empty one of the boxes, where \(0 \leq k < n\)?
3. Let $X$, $Y$, and $Z$ be independent random variables all of which have the exponential density $f(x) = e^{-x}, x > 0$. Find the probability of the simultaneous occurrence of the events that $X \leq Y$ and $X \leq 2Z$. 
4. Let $X$ and $Y$ be independent random variables with means both zero and common variance $\sigma^2$. Let $Z = 2X + Y$. Find $E(Z \mid X = x)$. 