Applied Mathematics and Statistics
Common Qualifying Examination Part B
in Computational Applied Mathematics

Spring 2011 (January)

(Closed Book Exam)

Please solve 3 out of 4 problems for full credit.

Indicate below which problems you have attempted by circling the appropriate numbers:

Part B: 1 2 3 4

NAME _____________________________

Start each answer on its corresponding question page. Print your name, and the appropriate question number at the top of any extra pages used to answer any question. Hand in all answer pages.

Date of Exam: January 31st, 2011
Time: 11:00 AM – 12:00 PM
B1. Consider the differential equation $y''(x) - 4y'(x) + 4y(x) = 3e^{2x}$.

a) Find its general solution using variation of parameters.

b) Find the particular solution that satisfies the boundary condition $y(0) = y(1) = 0$. 

B2. Solve the initial value problem

\[ x' = Ax + f(t), \quad x(a) = x_a, \]

where

\[ A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}, \quad f(t) = \begin{bmatrix} 4t \\ 1 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]
B3. A matrix $A \in \mathbb{C}^{m \times m}$ is normal if it $AA^H = A^H A$.

a) Show that if a matrix $T$ is both triangular and normal, then it must be diagonal.

b) Show that a matrix $A$ is normal if and only if it is unitarily diagonalizable, i.e., there is a unitary matrix $Q$ and a diagonal matrix $D$ such that $Q^H AQ = D$. 
Given $A \in \mathbb{R}^{m \times n}$, where $m \geq n$, assume it has full rank.

a) Show that
$$\begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} r \\ x \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$
has a solution where $x$ minimizes $\|Ax - b\|_2$.

b) Show that the largest singular value of the coefficient matrix
$$\begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix}$$
is greater than that of $A$. 