PROBABILITY THEORY  QUALIFYING EXAMINATION

Spring 2002

NAME: _________________________________

Instruction: Work three of the following four problems.

1. Let \( \{A_n\} \) be a sequence of sets. Show the Bonferroni’s inequality,

\[
P(\cap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n - 1).
\]

2. Suppose that \( X \) takes on one of the values 0, 1, 2. If for some constant \( c \), \( P(X = i) = cP(X = i - 1) \), \( i = 1, 2 \), find \( E(X) \).

3. Let \( X \) be a standard normal random variable, Find the PDF of the random variable \( Y = 2X^2 + 1 \).

4. Consider two independent random variables \( X \) and \( Y \) whose PMFs are given by

\[
p_X(x) = \begin{cases} 
    1/2 & \text{if } x = 0, 1, \\
    0 & \text{elsewhere,}
\end{cases}
\quad
p_Y(y) = \begin{cases} 
    1/2 & \text{if } y = 1, 2, \\
    0 & \text{elsewhere.}
\end{cases}
\]

Let \( R \) be the random variable that takes with equal probability either the value of \( X \) or the value of \( Y \). Let \( G \) denote the sum of six independent experimental values of \( R \). Find the mean and variance of \( G \).