180 Years of Market Draw Downs

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• An investment’s draw down behavior is an important element of its behavior.

• We will focus on a single market: the S&P 500 Total Return from 1835 to 2015 (Global Financial Data).

• Questions?
  o How can draw downs be modeled and analyzed?
  o How stable is this aspect of performance?
  o What insights can we develop examining the draw downs in two major markets over extended periods?
Recovery from Drawdowns

Gain Needed to Recover From a Loss

-10%: +11%
-20%: +25%
-30%: +43%
-40%: +67%
-50%: +100%

Return (%)
Market Drawdowns

Market Draw Downs Jan 01, 1835 through May 31, 2015
S&P 500 Total Return Index
Source: Global Financial Data
Market Drawdowns

Market Draw Downs Jan 01, 1835 through May 31, 2015
S&P 500 Total Return Index
Source: Global Financial Data
Max Drawdown Depth Distribution

Exponential-Gamma Mixture. Gamma distribution is a common distribution for a rate parameter, $0 < \lambda$:

$$\int_{0}^{\infty} \left( \frac{e^{-\lambda/\beta} \beta^{-\alpha} \lambda^{-(1+\alpha)}}{\Gamma[\alpha]} \right) (\lambda e^{-\lambda x}) d\lambda$$

Integrating out $\lambda$, the result is a variant of a Pareto distribution (Lomax):

$$f_{\Delta}[x] = \alpha \beta (1 + \beta x)^{-(1+\alpha)}$$
Log-Log Survival

\[ \text{Prob}[X > x] \approx L(x) \times x^{-\alpha} \]
Beta-Pascal Mixture. Pascal distribution is a common distribution for a probability parameter, 0 < p < 1:

\[
\int_0^1 \frac{(1 - p)^{\beta-1}p^{\alpha-1}}{B[\alpha, \beta]}((1 - p)^{k-1}) p \, dp
\]

Integrating out \( p \), the result is a variant of a Waring Yule distribution:

\[
\frac{\alpha (\beta)_{k-1}}{\Gamma[\alpha + \beta]}
\]

where \( (a)_n = \Gamma[a + n] / \Gamma[a] \)
Probability Plot

Drawdown Length Probability Plot, Jan 01, 1835 through Jul 31, 2018

BetaPrimeDistribution[1, 1.9361, 1, 0.0821954]
Log-Log Survival

Drawdown Survival Function, Jan 01, 1835 through Jul 31, 2018
Waring Yule Distribution[0.98077, 1.69615]

\[ \text{Prob}[X > x] \approx L(x) x^{-\alpha} \]
One spends about 75% in a drawdown state. And more than half the time in a major drawdown.
The simplest situation is to assume that the return process consists of independent identically distributed (i.i.d.) random variables.

Fairly simple arguments, supported by simulation studies, tell us that the depth of drawdown of an i.i.d. process will be exponentially distributed.

The data for market drawdowns are profoundly different.

Recall our questions?

- How can drawdowns be modeled and analyzed?
- How stable is this aspect of performance?
- What insights can we develop examining the drawdowns in an important market over an extended period?
Pre Great Depression
Post World War II
Post World War II

Drawdown Survival Function, Full Period vs. Post WW II

Survival Function vs. Drawdown Depth

- Full Period
- Post WW II
Fit Comparisons

Sampling Distribution Post WW II
S&P 500 Total Return Index

1,000 Runs of Sample Size 200
from 1951–2018 Data
Shaded Area 95% Coverage Interval

PDF

Tail Exponent (α)

0.0
0.2
0.4
0.6
0.8
1.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0

1835 – 2018
1835 – 1928
1951 – 2018
Conclusions

• Take the long historical view.
• Certain market characteristics are stable despite immense changes in science, technology, laws, and policies.
• Market max draw downs, as a driver of “regret”, may play a greater role than we realize.
• Conventional stress tests do not capture the true draw down behavior of financial markets.
• It would be a mistake to view the Great Depression as an “outlier”.
• Perhaps comparisons of different histories can be suggestive of what characteristics create more stable financial systems.